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Tunneling Radiation Characteristic of the Charged Particle from the Reissner–Nordström–anti de Sitter Black Hole

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Abstract Applying Parikh's semi-classical quantum tunneling method, the tunneling radiation characteristic of the charged particle from the event horizon of the Reissner–Nordström–anti de Sitter black hole is researched. The result shows the derived spectrum is not purely thermal one, but is consistent with the underlying unitary theory, which gives a might explanation to the information loss paradox and is the correct amendment to the Hawking radiation.

Keywords Charged particle · Reissner–Nordström–anti de Sitter black hole · Tunneling rates · Bekenstein–Hawking entropy

1 Introduction

"No hair" theorem, which indicated all the information of black holes would not be known apart from the mass, the charge and the angular momentum, was put forward by Feller etc. But, before long, Stephen Hawking made a striking discovery that the black holes can radiate particles and that the radiation spectrum is purely thermal one [1, 2]. It indicated the information can be carried by the radiation particles and the black holes will lose energy, shrink, evaporate and eventually disappear along with the particles being radiated. However this will induce the information loss paradox. In the past few decades, quite a few people was trying to overcome the information loss paradox but all of them failed. Ultimately, it was claimed that the problem could be solved in favor of the conservation of information by a conjectured AdS/CFT duality between string theory in anti-de Sitter space-time and a conformal field theory on the boundary of the anti-de Sitter space [3]. But there wasn't enough evidence to prove it. In addition, Hawking treated the radiation process of the particles as the quantum tunneling effect, but there was not any potential barrier appeared. And it was also not clear to the mechanism about the cause of the quantum tunneling potential barrier.

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Recently, a new semi-classical method, taking energy conservation and the unifixed background space time into account, was developed by Kraus and Wilczek and elaborated upon by Parikh and Wilczek [4]. They put this method into the research on the Hawking radiation of the static Schwarzschild black hole and Reissner–Nordström (R-N) black hole, and achieved a great deal of success [5–7]. They pointed out definitely the tunneling potential barrier is produced by self-gravitation interaction of the outgoing particles; thereby the cause of the mechanism corresponding to the tunneling potential hill has been resolved. In addition, the Painlevé coordinate that is well behaved at the event horizon was introduced. The result shows that Hawking radiation of the black hole is not strictly thermal, which is consistent with the underlying unitary theory. So this gives the information loss paradox a might explanation.

Extending Parikh's method, quite a few papers about the Hawking radiation of the uncharged massless particles from the black holes in de sitter space-time have been researched and all of these results are in accordance with Parikh's [8–12, 14]. However, the general black holes are charged, thus the radiation particles corresponding to them should be also charged and the electro-magnetic energy should be taken into account during implementing the energy conservation. In this paper, the tunneling radiation characteristic of the charged particle in anti-de Sitter space-time from the event horizon of Reissner–Nordström anti-de Sitter (R-N AdS) black hole is discussed. R-N AdS black hole only has one horizon (the event horizon EH, r_+) and is determined by three parameters: EH(r_+), the AdS radius R (which is related to the negative cosmological constant by $R^2 = -\frac{3}{\Lambda}$) and the charge parameter Q. The result shows the derived spectrum is not precisely thermal one, and therefore the information loss paradox has been gotten a might explanation.

The remainder of this paper is organized as following. In the next section, we perform Painlevé coordinates transformation which is well behaved at the horizons and obtain the radial geodesic of the charged massive particles. In our paper, the radial geodesic is different from that of the uncharged massless, which is the phase velocity of the particles. In Sect. 3, the tunneling radiation characteristic of the charged particle is discussed via the R-N AdS black hole. In the last section, some conclusion and discussion are given.

2 Painlevé Coordinate Transformation and Radial Geodesic of Charged Particle

According to [15], R-N AdS black hole is given by

$$ds^2 = -h dt_R^2 + h^{-1} dr^2 + r^2 d\Omega^2, \tag{1}$$

with the mass

$$M = \frac{1}{2}\left(r_+ + \frac{r_+^3}{R^2} + \frac{Q^2}{r_+}\right), \tag{2}$$

and the electromagnetic potential

$$A_\mu = (A_t, 0, 0, 0), \tag{3}$$

where

$$A_t = \frac{Q}{r} \quad \text{and} \quad d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2,$$

$$h = 1 - \frac{r_+}{r} - \frac{r_+^3}{R^2 r} - \frac{Q^2}{r_+ r} + \frac{Q^2}{r^2} + \frac{r^2}{R^2},$$

R is the AdS radius which is related to the cosmological constant by $R^2 = -\frac{3}{\Lambda}$, and r_+ is EH of the R-N AdS black hole. However, the coordinate singularity in (1) is inconvenient to research the tunneling radiation of the charged particle and we should find out a coordinate system well behaved at EH, so we introduce the Painlevé coordinate transformation [16]

$$dt = dt_R + f'(r)dr, \tag{4}$$

and have

$$ds^2 = -hdt^2 - 2hf'(r)dtdr + [h^{-1} - hf'^2(r)]dr^2 + r^2 d\Omega^2. \tag{5}$$

Considering the anti-de-Sitter-Painlevé coordinates have the same geometry as constant time slices of a global anti-de Sitter metric, we order

$$h^{-1} - hf'^2(r) = \left(1 + \frac{r^2}{R^2}\right)^{-1} = P. \tag{6}$$

So the line element of the Painlevé-R-N AdS black hole is rewritten as

$$ds^2 = -hdt^2 + 2\sqrt{1 - hP}dtdr + Pdr^2 + r^2 d\Omega^2, \tag{7}$$

where the coordinate singularity has been eliminated. Furthermore, there are other attractive features: the event horizon and the infinite red-shift surface are coincident with each other; the metric satisfies Landau's condition of the coordinate clock synchronization and there exists a Killing vector ∂_t, so the derived space-time line element is stationary. All these properties provide convenience to research the Hawking radiation of the charged particle via tunneling from the event horizon.

We know the R-N AdS black hole is charged and massive, the corresponding radiation particles should be also charged and have mass, so the geodesic of the light-like particles can not be relied here, and we have to find out the radial geodesic corresponding to the charged massive particles. According to [13, 17], the radial geodesic of charged massive particles is the phase velocity of particles, which can be expressed as

$$\dot{r} = v_p = -\frac{1}{2}\frac{g_{00}}{g_{01}} = \frac{h}{2\sqrt{1 - hP}}. \tag{8}$$

3 The Hawking Radiation of Charged Particle via Tunneling from EH

Considering the ADM mass and the charge of the total space-time are held fixed and that of the black hole are allowed to vary, when particles with energy and charge tunnel out, the black hole will shrink and the location of EH will change also. So when energy conservation and charge conservation are taken into account, if the particle with energy ω and charge q emitted out, the mass and charge parameters in (3), (7) and (8) will be replaced by $M - \omega$ and $Q - q$. Here, r_{+i} and r_{+f} denote the locations of the event horizon before and after the charged particle tunnels out from the event horizon respectively. Now, the line element of the black hole has to be rewritten by

$$ds^2 = -h'dt^2 - 2\sqrt{1 - h'P}dtdr + Pdr^2 + r^2 d\Omega^2, \tag{9}$$

with the outgoing geodesics of the charged particle

$$\dot{r} = \frac{h'}{2\sqrt{1-h'P}}, \tag{10}$$

where

$$h' = 1 - \frac{r_{+f}}{r} - \frac{r_{+f}^3}{R^2 r} - \frac{(Q-q)^2}{r_{+f} r} + \frac{(Q-q)^2}{r^2} + \frac{r^2}{R^2}.$$

In the line element (9), due to the coincidence between the event horizon and the infinite red-shift surface, thus the geometrical optics limit is reliable here, and we can use particle language to describe the tunneling radiation. Applying WKB approximation [18, 19], the formula between the tunneling rate and the imaginary part of the particle action satisfies

$$\Gamma \sim e^{-2\,\mathrm{Im}\,S}, \tag{11}$$

where the action of the particle is

$$S = \int_{t_i}^{t_f} L dt, \tag{12}$$

where L is Lagrangian function. When the charged particle tunnels out, the effect of the electro-magnetic field should also be taken into account, so the matter-gravity system must consist of the black hole and the electro-magnetic field outside the black hole. As $-1/4F_{\mu\nu}F^{\mu\nu}$ is the Lagrangian function of the electro-magnetic field corresponding to the generalized coordinate A_μ, so A_t is an ignorable coordinate. To eliminate the freedom, the imaginary part of the particle action should satisfy as follows

$$\mathrm{Im}\,S = \mathrm{Im}\int_{t_i}^{t_f} (L - P_A \dot{A}'_t) dt = \mathrm{Im}\left[\int_{r_{+i}}^{r_{+f}} \int_0^{p_r} dP'_r dr - \int_{A_{ti}}^{A_{tf}} \int_0^{p_A} dP'_A dA'_t\right], \tag{13}$$

where P_A is the canonical momentum conjugated to the electromagnetic potential A_t. r_{+i} and r_{+f} can be seen as two turn points of the potential barrier, and the distance between them depends on the energy and the charge of the tunneling particle. For proceeding with an explicit calculation, using Hamilton canonical equation,

$$\begin{aligned} \dot{r} &= \left.\frac{dH}{dP_r}\right|_{(r;A_t,P_A)}, \qquad dH|_{(r;A_t,P_A)} = -d\omega, \\ \dot{A}_t &= \left.\frac{dH}{dP_A}\right|_{(A_t;r,P_r)}, \qquad dH|_{(A_t;r,P_r)} = -\frac{Q-q}{r} dq. \end{aligned} \tag{14}$$

Substituting (10) and (14) into (13), we have

$$\begin{aligned} \mathrm{Im}\,S &= \mathrm{Im}\int_{r_{+i}}^{r_{+f}} \int_{(M,E_Q)}^{(M-\omega,E_{Q-q})} \frac{dr}{\dot{r}} \left(dH|_{(r;A_t,p_A)} - dH|_{(A_t;r,p_r)}\right) \\ &= -\mathrm{Im}\int_{r_{+i}}^{r_{+f}} \int_{(0,0)}^{(\omega,q)} \frac{2\sqrt{1-h'P}}{1 - \frac{2(M-\omega')}{r} + \frac{(Q-q')^2}{r^2} + \frac{r^2}{R^2}} \left(d\omega' - \frac{Q-q'}{r} dq'\right), \end{aligned} \tag{15}$$

where E_Q and E_{Q-q} represent the energy of the electromagnetic field, doing the ω' and q' integral firstly and getting

$$-\int_{(0,0)}^{(\omega,q)} \frac{2\sqrt{1-h'P}}{1-\frac{2(M-\omega')}{r}+\frac{(Q-q')^2}{r^2}+\frac{r^2}{R^2}}\left(d\omega' - \frac{Q-q'}{r}dq'\right) = -i\pi r. \tag{16}$$

Switching integral and finishing it, we can obtain

$$\operatorname{Im} S = \operatorname{Im}\int_{r_{+i}}^{r_{+f}} (-i\pi r)dr = -\frac{\pi}{2}\left(r_{+f}^2 - r_{+i}^2\right). \tag{17}$$

So we have

$$\Gamma \sim e^{-2\operatorname{Im} S} = e^{-\pi(r_{+f}^2 - r_{+i}^2)} = e^{\frac{1}{4}(A_f - A_i)} = e^{\Delta S_{BH}}, \tag{18}$$

where $A_i = 4\pi r_{+i}^2$ and $A_f = 4\pi r_{+f}^2$ are the areas of EH before and after the charged particle tunneling out, $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(M-\omega, Q-q) - S_{\mathrm{BH}}(M, Q)$ is the change of the Bekenstein–Hawking entropy. So the derived spectrum is not purely thermal one, and the result which is consistent with the underlying unitary theory and provides a might explanation to the information loss paradox.

4 Conclusion and Discussion

The above discussion on the tunneling radiation of the charged particle from event horizon of the black hole in anti-de Sitter space-time shows the derived spectrum deviate from precisely thermal one, which gives the information loss paradox a might explanation, and provides a correct modification to the Hawking thermal spectrum. In addition, in quantum mechanics, the tunneling rate should be expressed as

$$\Gamma(i \to f) \sim |M_{fi}|^2 \quad \text{(the phase space factor)},$$

where $|M_{fi}|^2$ is the square of the amplitude for the tunneling process, and the phase-space factor is given by summing over the final states and averaging over the initial states. As the number of the initial/final states corresponds to the initial/final entropy, so the tunneling rate satisfies

$$\Gamma \sim \frac{e^{S_{\mathrm{final}}}}{e^{S_{\mathrm{initial}}}} = e^{\Delta S}. \tag{19}$$

Therefore, our result is consistent with (19) and satisfies the underlying unitary theory.

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